USN

Fifth Semester B.E. Degree Examination, June-July 2009
Modern Control Theory

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Explain the advantages and disadvantages of selecting (i) Physical variables (ii) Phase variables and (iii) Canonical variables for state-space formulation of control systems. **(06 Marks)**

b. Fig.1(b) gives a schematic diagram of an armature controlled D.C. servomotor. Assume the motor is operating in the linear region. Determine the state equations in the vector-matrix form for the state variables $X^T = [\theta \;\; \dot{\theta} \;\; i_a]$.

M.I. of motor and load = J ; Back emf constant = k_b ; Coefficient of friction of motor and load = f ; Motor torque constant = k_T ; θ = output. Also draw the block diagram. **(07 Marks)**

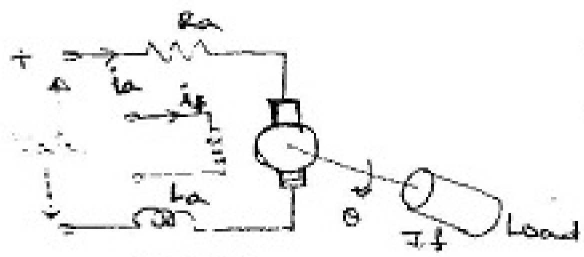

Fig.1(b)

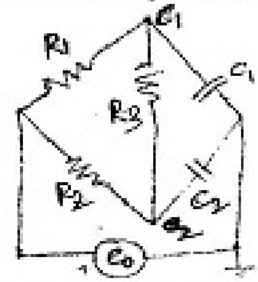

Fig.4(a) [Question on P2]

c. Linearise the state equation $\dot{X} = f(X, U)$ of a general time-invariant system for small variations about an equilibrium point (X_0, U_0) by expanding it into Taylor series. Neglect terms of second and higher order. **(07 Marks)**

2 a. Consider the transfer function defined by $\dfrac{Y(s)}{U(s)} = \dfrac{b_0 s^n + b_1 s^{n-1} + \ldots\ldots\ldots b_{n-1}s + b_n}{(s+p_1)(s+p_2)\ldots\ldots\ldots(s+p_n)}$

where $p_i \neq p_j$. Derive the state space representation of this system. Also give the block diagram representation of the system. **(10 Marks)**

b. Obtain a transfer function representation of the system described by the state space model $\dot{X} = Ax + Bu$ and $y = cx$

where $A = \begin{bmatrix} 0 & 1 & 0 \\ -1 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ and $C = [0 \;\; 0 \;\; 1]$ **(07 Marks)**

c. What is a generalized eigen vector? Explain. **(03 Marks)**

3 a. Consider the state equation $\dot{X}(t) = \begin{bmatrix} 0 & 1 \\ -8 & -6 \end{bmatrix} X(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$

The system is initially at $X(0) = [1 \;\; 0]^t$.

i) Compute state transition matrix by Cayley – Hamilton method. ii) Find the states for any time t with no input. iii) Find the states for any time t with a unit step input. **(15 Marks)**

b. A system is described by the state equation $\dot{X} = Ax + Bu$ where X is an n – dimensional state vector and u is r – dimensional input vector. Assuming that all the eigen values of A are distinct, state the necessary and sufficient condition (due to Gilbert) for complete state controllability. Derive the necessary equations. **(05 Marks)**

4 a. Consider the electrical network shown in Fig.4(a). By selecting the capacitor voltages as state variables, form the state equations of the network. Therefrom, form the controllability matrix and check whether the system is controllable under balance condition. [Fig. on P1] **(12 Marks)**

b. State the properties of state transition matrix. **(03 Marks)**

c. Compute the model matrix if the system matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 3 & 0 & 2 \\ -12 & -7 & -6 \end{bmatrix}$ **(05 Marks)**

PART – B

5 a. Compare the pole placement method of design of a CISO control system with the conventional approach. **(04 Marks)**

b. A simgle – input linear time invariant system is described by n^{th} order state differential equation $\dot{X} = Ax(t) + Bu(t)$. Prove that the closed – loop poles of the system can be arbitrarily assigned if the system is completely state controllable. **(08 Marks)**

c. Consider a system defined by $\dot{X} = Ax + Bu$ and $y = Cx$

where $A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$.

It is desired to place the eigen values at – 3 and – 5 by using a state – feedback control u = - kx. Determine the necessary feedback gain matrix K and the control signal u. Apply Ackerman's formula. **(08 Marks)**

6 a. Define (i) Full – order state observer (ii) Reduced order state observer (iii) Regulator systems. **(06 Marks)**

b. Draw the block diagram of 'Luenberger state observer' and write the state equation in terms of the estimated state vector. **(06 Marks)**

c. Explain the following nonlinear phenomenon
i) Frequency – amplitude dependence ii) Multi-valued responses and jump resonances. **(08 Marks)**

7 a. What are singular points? Classify. **(06 Marks)**

b. What are limit cycles? Explain the limit cycle behavior of non-linear systems by considering Vander Pol's differential equation. **(07 Marks)**

c. Explain in detail the various steps involved in the construction of phase trajectory by Delta method. **(07 Marks)**

8 a. Check the sign definiteness of following quadratic equation.

$$V(x) = [x_1 \; x_2 \; x_3] \begin{bmatrix} 4 & 1 & 1 \\ 1 & 2 & \frac{1}{2} \\ 1 & \frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$ **(04 Marks)**

b. State the Liapunov's main stability theorem. Examine the stability of the system described by $\dot{x}_1 = x_2 - x_1(x_1^2 + x_2^2)$

$\dot{x}_2 = -x_1 - x_2(x_1^2 + x_2^2)$ **(08 Marks)**

c. Determine the stability of the system $\dot{X} = Ax$ where $A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$ by Liapunov's theorem and hence determine a suitable Liapunov's function. **(08 Marks)**

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